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GEOMETRIC STEREOGRAMS—A DEVICE FOR MAKING SOLID GEOMETRY TANGIBLE TO THE AVERAGE STUDENT.

By Walter Francis Shenton.

In all the gamut of mathematical studies there is not another subject which has given teachers so much concern as Solid Geometry. Time and thought and energy to the n-th power have been given to the preparation of the subject matter, and to the revision of the course so as to make it a logical successor to Plane Geometry, but after a class has been carefully taught by the best available methods it is almost heart-rending to see how many members of the class do not have even the fundamental ideas of space relation correctly fixed in their minds.

In order to make these space relations clear the teacher has had for his resources up to the present time such models as may be bought or made for demonstration and use in the class room. Wood and plaster and silk strings, to say nothing of cards and wires have been put together in the various shapes that the theorems of solid geometry require for demonstration and from them the students have tried to understand the meaning of the plane representation of the solid figure. Of these models we have nothing to say in any unkind way, for they were the means of giving us our first complete notion of the complex relations of lines and planes in space, and more than once a desk top and a number of pencils have illustrated to the writer certain relations that he could not otherwise fully comprehend. But at the very best they are clumsy and friable and the student must be under the watchful eye of the teacher always when he is using them lest damage result to the models. But this is not their worst fault. It consists in the fact that they are not available to the individual student when he is preparing his work. Even supposing that they do help him straighten out the tangle he had gotten into during his study of the night before, how much better it would have been if he had never gotten into the tangle!

This condition of affairs has made it desirable that each student should have a set of simple models of his own which he may keep with him when he studies and thus form correct ideas from the start. To ask that this may be literally fulfilled is too evidently futile to be worthy of further consideration. However, after a fashion this very thing has been done by the firm of Underwood and Underwood whose name is common property at the present in connection with the finest photographs of current events that are published, and whose travel photographs in stereoscopic form now accord many of us the privilege of seeing in a very realistic way, scenes of countries whose soil we may never hope to tread. They have met the solid geometry need to some small degree by a series of illustrations for various solid figures which when viewed through their stereoscope give relief. Their method, however, has proved too bulky and expensive for the average student.

However, in the past quarter of a century there has been working out an old idea in new form, and it is the purpose of this paper to give briefly the history, theory, and results of this method. In 1853 there appeared in the *Poggendorf Annalen,* an article by W. Rollman (*90,* p. 187) in which was set forth a method of using a two-colored figure viewed with a pair of two colored glasses to obtain the effect of relief. D'Almeida used this method in 1858—probably independently re-discovered it—in the form of viewing with bi-colored glasses a two-colored figure thrown on the screen by a lantern and this same method was also developed by Molteni. However, it did not come before the public eye to any considerable extent until 1894 when L. D. Duhauron had a modification of this method patented in this and other countries. His method took cognizance of the work of the previous inventors and was based entirely on the idea of producing actual drawings, half-tones, or any other forms of actual print, which were to be viewed with a pair of colored glasses as will be explained shortly. Although he produced quite a collection of prints marketed as "Plastographical Views of the World," the method did not seem to have made a serious impression upon the mathematical world until the Cambridge International Congress of Mathematicians in 1912, when M. H. Richard and M. H. Vuibert presented a set of "Anaglyphes" as they called them. The figures showed applications to ordinary solid geometry, spherical geometry, descriptive geometry, crystallography and physics, and were very well received. In the fall of the same year they published a little brochure entitled "Les Anaglyphes Geometriques" which described the theory of the process and gave some twelve pages of figures which when viewed through the glasses which accompanied the text gave plastographic effect.

This little book set the writer seriously to work as he had been much interested in the making of models and this seemed a very practical way out of the general difficulty. The results of the work of the past three years are presented briefly in what follows.

The physiological effect of seeing a solid figure in space is a comparatively simple thing. Each eye, acting as a small camera, sees a small reproduction of the body in front of it and within its range of vision. This reproduction is essentially a section of the pencil of rays passing through a focal point somewhere near the pupil of the eye. As the two pupils are at a distance of about seven centimeters from each other, each eye sees the figure from a different point of view and so each of the reproductions is a section of a different pencil of rays. If now, a cube, say, be set on a sheet of paper and viewed from a fixed position of the eyes and the eyes closed alternately while the section of the ray-pencil is sketched on the paper for each eye, we will have a peculiar double trace of the figure, one half of which corresponds to the section of the pencil of rays for each eye by the plane of the sheet of paper. Since each section of this pencil will give the same effect to the eye concerned as the original solid, if we can in any way separate the two figures so that each eye sees only the section that belongs to it, we shall have the same physiological effect as would be caused by the actual solid.

This may be accomplished by calling into play the following physical phenomenon. If a line of pure color be drawn on a sheet of white paper and then viewed through a plane screen of the same color, that is, which transmits rays of light having the same frequency as those given off by the colored line, the line will apparently disappear. What actually happens is that the whole white sheet appears through the screen to be of exactly the same color as the line, which is then lost in the common tone of the background. If now we view a line of contrasting color through the screen, it will not reflect any rays of the frequency that will pass through the screen, and so will appear through it as a black line on a colored ground.

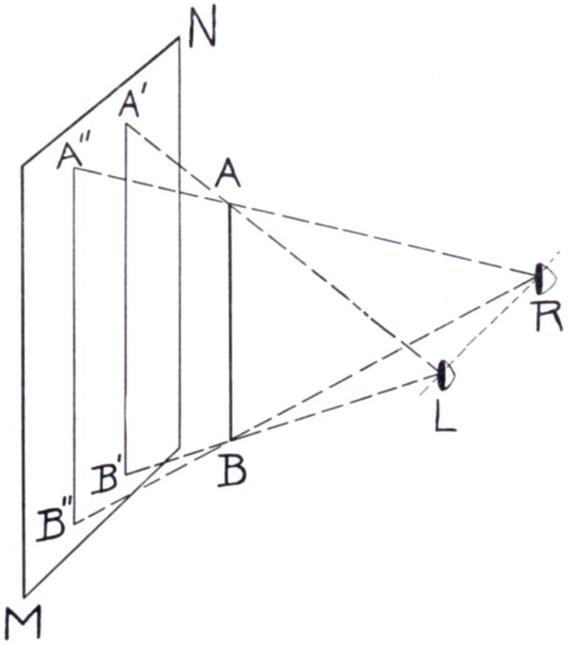

Fig. 1.

Returning now to our original proposition. If we were to draw on a sheet of white paper in green, for example, the section of a figure corresponding to what the right eye would see, and in complementary red the section corresponding to that for the left eye, then each eye would see its own figure in black on a colored ground, provided that the right eye were screened with red and the left eye with green. By choosing the red and green as nearly complementary as may be in practice, the effect to the two eyes due to the mixing of the two backgrounds will be a grayish white, and the effect of the two black figures will be that of a solid in relief. This is the physical theory of the method.

Let us now turn our attention to the methods at hand for making the drawings, which methods are in practice somewhat hard to develop. First consider the very simplest case, when the object which we wish to draw, say a vertical straight line in space, lies between the observer and a plane which is vertical to the earth's surface and parallel to the line joining the pupils of the eyes of the observer. In this case the two figures desired are the two traces made in the given plane by the two planes which are determined by the given line and the pupil of one eye of the observer. In Fig. 1 suppose that AB is the given vertical line in space and that MN is the vertical background plane, while the eyes are denoted by their initial letters L and R. Then $A'B'$, the intersection of the triangle LAB with MN, is the line in our figure that corresponds to the left eye; and $A''B''$, the intersection of the triangle RAB with MN, is the line which corresponds to the right eye. However, this requires that we have the actual line in space, and we have actually used this method more or less extensively in preparing some figures when we had the actual models at hand in convenient shape and size. When the figure becomes too complex, or no models are at hand, we must use an extension of this method. In the designing of figures we make use of the three orthogonal projections of the solid in the three planes of reference, from which, by devices that are beyond the scope of this paper we are always able to produce a correctly drawn figure without even having visualized the solid.

The method just outlined is such as we would have to use for figures which are to be projected on a vertical screen by means of such apparatus as a reflectoscope, and viewed by a class, or for figures which we wish held vertically before the observer. A rather more natural way to produce the drawings is to make them appear to rise from a sheet of paper laid horizontally on a desk and viewed so that the line of sight makes an angle of say 45° with the vertical. As this causes the figures to have the actual appearance of standing up on the sheet it produces a very startling effect. The general scheme for producing the figures of a vertical line viewed in this way is shown in Fig. 2.

One of the very interesting things in connection with drawings made by the first method is that if the figures are not too complex, when we view them with the glasses reversed they appear to penetrate the paper instead of coming out from it as is normal. By this means what would normally appear as a frustum of a cone standing out from the sheet, its base on the background, becomes, when the glasses are reversed, a much longer frustum hollowed out of the sheet. Many persons have difficulty in seeing this because they cannot persuade themselves that the figure penetrates the paper as it seems to do in this case.

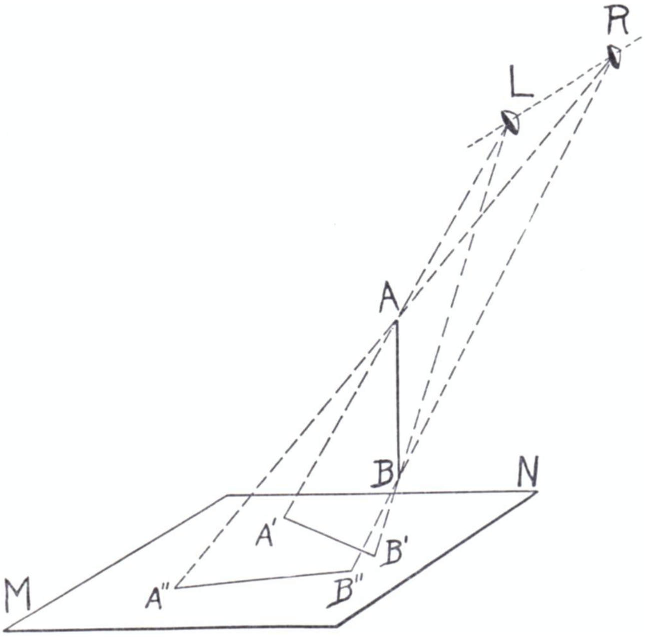

FIG. 2.

It has been a rather interesting observation that almost any school boy will be able to raise the figures at once and describe exactly what he sees there, while some of the more perfectly trained mathematicians of my acquaintance have difficulty in making the figures seem real. In any instance where there is difficulty in seeing the figure clearly in relief, it can almost always be overcome by asking the spectator to move his head from side to side, or else to move the drawing slowly from right to left and back, in which case the figures will appear to rise from the paper and follow his eyes as they move. This peculiar moving of the figures is one of the interesting things in connection with these drawings.

Quite a number of technical difficulties have been met in trying to make a commercial proposition out of this theory. Paramount among them has been the difficulty of getting inks and screens for the drawings, which will behave as we have indicated is necessary. Having overcome this obstacle by dyeing our own screens with the same dyes that we use for our drawing inks, we were face to face with a more serious trouble in the nature of being unable to get printer's inks which would work as we desired. After about two years of experimenting along this line we have been able to produce prints which are even better than our original drawings. In addition we have been able to find a limited supply of gelatine screens of the colors desired, so that the problem seems to have been solved, technically speaking.

It is our purpose to produce a set of drawings to cover all the figures of solid geometry, lettering them to agree with one or several of the texts now in most common use. The larger number of these will be drawings like the sample that we have submitted here, but such figures as the sphere and other curvilinear solids will probably be presented by means of half-tones from photographs of actual models.

We believe that figures presented in this way will be an invaluable addition to the teaching equipment now accessible for solid geometry and that they will stimulate the students to a better understanding of one of the dark places in mathematics. This method possesses the following very distinct advantages over any other similar method that has been suggested:

1. It is just as easy on the eyes as looking at the actual space figures would be.

2. It is self-contained. A small book or portfolio with an envelope in the cover to contain the glasses would be the sum total of apparatus required.

3. The simplicity of the apparatus will make the process inexpensive. So inexpensive, we believe, that each student can afford to have a set of his very own.

4. The figures with their bold relief will immediately fascinate

the average student, especially when he discovers that he can make them bend for him when he moves his head from side to side. I have seen this very vividly illustrated by the delight that both teachers and students have shown when they made this latter discovery.

5. An actual model bearing the same letters as the black-and-white plane figure in the text, used in the student's room, will not only enable him to understand the proposition, but will also make it possible for him to understand the "stupid" figure in the text.

Because of these advantages we claim that we have here a novelty which is destined to play a very large part in the teaching of solid geometry to the average and below-average student.

Note.—A limited supply of prints illustrating the proposition, ***The volume of any parallelopiped is equal to the product of its base and altitude,*** together with colored glasses, may be obtained at the rate of twenty-five cents for one print with pair of glasses. Address,

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JOHNS HOPKINS UNIVERSITY,
BALTIMORE, MD., April 1, 1915.